$$H_{1c} = |\rho_k|^{-1}[(\tfrac{1}{2}\omega - \omega_k)^2 + \eta_k^2]^{1/2} . \qquad (9)$$

In the case of Madsen and Tanaka[3] there is not a perfect agreement with the result of other theories, whereas in our case Eq. (9) is exact. The instability criterion (8) indicates the intensity of the excitation for which the Green's-function series does not converge. This criterion should also be applicable to determine the threshold condition in other nonlinear effects, such as the stimulated Raman or Brillouin scattering.

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Note on Gadzuk's Theory of Field-Induced Tunneling

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Recently, Gadzuk[1,2] presented theoretical studies concerning band-structure effects on field-induced tunneling from metals and resonant tunneling through atoms absorbed on metal surfaces. The calculation of the matrix elements necessary for comparison with experiment in these papers appears to be seriously in error and, at least in the second paper, to cast doubts on the usefulness of these calculations for comparison with the experimental work of Plummer and Young.[3] The basic error is the statement $j_l(ikr) = i^l j_l(kr)$. That this is incorrect can be seen clearly by considering the case $l=0$ where $j_0(x) = x^{-1}\sin x$. The matrix elements are easily recalculated by replacing $j_l(x)$ by $(\pi/2x)^{1/2} I_{l+1/2}(x)$ wherever it occurs. Thus, for example, Eqs. (35a) and (35b) of Ref. 2 should read

$$I_1 = \frac{\Gamma(7)}{k^8(\beta_s^2 - 1)^7}(7\beta_s^6 + 35\beta_s^4 + 21\beta_s^2 + 1) ,$$

$$I_2 = \frac{3\Gamma(8)}{k^{10}(\beta_s + 1)^8} + \frac{\Gamma(9)}{2k^{10}}[(\beta_s - 1)^{-9} - (\beta_s + 1)^{-9}] .$$

Hence, for $k \approx 1.2$ Å^{-1}, $a_s \approx 1.5$ Å^{-1}, $I_1(\text{corr})/I_1(35\text{a}) \simeq -25$, $I_2(\text{corr})/I_2(35\text{b}) \simeq 30\,000$.

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Reply to Comments on a Theory of Field-Induced Tunneling

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In the preceding paper,[1] Glasser has pointed out an oversight in some manipulative details leading to approximate expressions in this writer's theoretical studies of field-induced tunneling.[2,3] However, the mishap is not as disastrous as it might first appear. In all cases in which the tunneling matrix elements were calculated, the physically significant results were presented in the form of ratios of matrix elements. Thus the errors are of order unity rather than of order 30 000 which should not be too important since only relative strengths and positions of structure in the enhancement factors were of interest. Since the models upon which calculations were performed were sufficiently crude, the resulting numerical discrepancy should not be regarded as physically meaningful. The calculations were purely model calculations to illustrate physical points, and the importance of the numerical agreement between theory and experiment was simply to show that experimental field-emission and resonance tunneling data could be analyzed in a reasonably straightforward manner by choosing realistic parameters characterizing the interacting atom-metal system.

It was the hope that the simple model calculations would point out a new physical effect and stimulate further more realistic calculations. In Ref. 2, the fact that d-band tunneling is significantly lower than s-band tunneling was illustrated. As a result of this observation, Politzer and Cutler[4] have since confirmed this point, improved the model, and calculated numbers which are in qualitative accord with those obtained in Ref. 2.

The key spectroscopic results in the resonance tunneling analysis,[3] namely, that the ns level widths 2Δ are about 1 eV and the $(n-1)d$ level widths are about 0.1 eV in adsorbed alkaline-earth atoms, remain unaffected by Glasser's comments. The tunneling theory presented in Ref. 3, in which it is shown how experimental results can be analyzed in terms of level-width parameters, is in accord with other resonance tunneling theories.[5] Thus if we regard $2\Delta=\Gamma$ as a fitting parameter and do a fit to the data of Plummer and Young[6] with the tunneling theory result, Eqs. (13) and (19) of Ref. 3, then similar values for Δ as calculated[3] will obtain.

In conclusion, although the methods of obtaining the matrix elements in Refs. 2 and 3 must be revised as indicated by Glasser,[1] when this is done it is felt that the altered results will not be sufficiently different from the reported ones to make it worth doing.

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Comments on the Upper Critical Field of Type-II Two-Band Superconductors

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The justification for the existence of interband coupling between the pair wave functions Δ_s^* and Δ_d^* for two-band superconductors in the dirty limit is explained in greater detail.

In two previous papers[1,2] by the present author, a model two-band superconductor in the dirty limit was studied. It is found that even without interband BCS coupling, the pair wave functions $\Delta_s^*(\vec{x})$ and $\Delta_d^*(\vec{x})$ (for the s band and the d band, respectively) can be coupled through vertex corrections due to interband impurity scattering. As a result, we find that in the temperature region between the s-band transition temperature and the d-band transition temperature, the upper critical field is uniformly lowered by the interband impurity scattering (see I) and in the temperature region just above zero, $T \gtrsim 0$, the upper critical field for a superconductor with both bands in the superconducting phase is higher than that for a superconductor with d band in the superconducting phase and s band in the normal phase (see II). In a recent paper by Sung,[3] it is asserted that the calculation performed in I is not correct, and his reason is that he cannot think of any justification of Eqs. (I34) and (I35), which couple the pair wave functions $\Delta_s^*(\vec{x})$ and $\Delta_d^*(\vec{x})$ through interband impurity scattering. As a matter of fact, his assertion is not correct. A justification in the spirit of the Feynman-diagram technique for obtaining vertex corrections[4] is given in I. For the convenience of the readers of these papers, in this note we shall explain this in greater detail.

We start with Eqs. (I31) and (I32),

$$\Delta_s^*(\vec{x}) = g_s T \sum_\nu \int d^3l \, \langle G_{0ss}^A(\vec{1}, \vec{x}; -z_\nu) \times G_{0ss}^A(\vec{1}, \vec{x}; z_\nu) \, \Delta_s^*(\vec{1}) \rangle_{\text{vert. corr.}}$$